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WAGE AND STRIKES DETERMINATION IN AN ECONOMY WITH TWO TRADE UNIONS. A DIFFERENTIAL GAME APPROACH.

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INTRODUCTION

The process of wage determination is one of the most important institutional features of an economy. It determines the response to exogenous shocks and affects the long run perfomance as far as employment is concerned. For most western European countries the role of unions in that process is very important, regardless the size of the unions itself. It has become fashionable the explicit consideration of the unions both in theoretical and empirical macroeconomics. In this paper, a very particular issue of a unionized labour market is analyzed; We shall study an economy with several unions, and consider the different wage/employment outcomes wich result from alternative strategic behaviour of those (rational) unions. The model adopts a differential game format in wich each union choses its policy instrument, taking in account the (passive) response of the employer (through the labour demand) and the existence of other union(s) operating in the same firm. The state variable is the wage level; its dynamic is driven by the instruments choosen by the union plus some exogenos factors wich determine the relative bargaining power of firm and unions. The dynamic constraint of the model is rather ad hoc, it may be interpreted as coming up from a bargaining setting under uncertainty as is loosely described somewhere else (Andrés (1985)). A quick sketch of the process is given below.

Wages are determined by long term contracts wich are negotiated at a point in time. This fact, and the difficulty in finding meaningful variables causing wages in an econometric sense (Ashenfelter and Card (1983)), has led most empirical studies of the labour market to consider wages as reasonably exogenous in the determination of other macroeconomic variables such as employment [illegible]. When wages are seen as the result of a process of bargaining between firms and unions, things are different. Wages are no longer exogneous and the fact that no single party can decide about them, makes room for some strategic behaviour. Negotiations take place at a point in time whereas the other decisions (such as employment, [illegible], etc.), are [illegible]

Nevertheless there is a mutual influence between the two types of decissions. The outcome of the negotiation, i.e. wages, will affect the performance of the firm and the utility of the unions, the everyday production and pricing choices and the union's policy instrument will affect the negotiation and hence the wage itself.

The joint determination of prices, wages, employment and union's instrument, may then be put in a dynamic optimization format. The model considers then an explicit optimizing behaviour for both unions and fims, whereby they choose their respective policy instruments, subject to the constrain of the wage dynamics. It is this las bit the one wich is not explicitely derived from the bargaining process, for no formal model is supplied. The equation is taken as a linear approximation to a wage equation coming up from a bargaining setting (loosely speaking) and following Cramton (Cramton (1983)) it includes the following variables:

$$\dot{W}(t) = F\left[w(t),\ C_u(t),\ C_F(t),\ Z_U(t),\ Z_F(t)) \right] \qquad (1)$$

where, w (t) is the (real) wage, C (t) stands by the costs of delaying the agreement and $Z_i(t)$ the exogenous factors behind the desired wage settlement for bothe the union and the firm. In a bargaining model under uncertainty C(t) will include strikes (as the means by wich the union makes the delay costly for the firm) and inventories (as the corresponding firm's defense against industrial action). Some econometric work carried out by author (Andres (1984)) also supports the idea of bringing those two variables in an structural wage equation. A four dimensional vector autorregresion analysis (with wages, prices, employment and interest rates) for the U.K., confirmed the Ashenfelter and Card's results for the U.S.(Ashenfelter and Card (1982)). The similarity found between the causality patterns for the two countries was in fact outstanding. Wages, in particular, follow a low order autorregressive process and no other variable adds significance to the equation with lagged wages in.

The consideration of strikes and inventories produces a dramatic change in those results. Both variables (jointly and otherwise) strongly cause wages in an econometric sense. On top of this the general unrestricted vector autorregressive model, displays most long run properties one would like to find: long run positive (negative) coefficient of strikes (inventories) on wages, long run neutrality of nominal variables in the demand for labour, long run zero elasticity, of inventories in the demand for labour, and so on. For all this (1) can be written as.

$$\dot{w}(t) = -\alpha_1 w(t) + \alpha_2 S(t) + \alpha_3 I(t) + \alpha_4 Z_U(t) + \alpha_5 Z_F(t) \qquad (2)$$

where: $S(t)$ = some strike variable.
$I(t)$ = inventories.

The general framework for the determination of prices $(p(t))$ employment $(n(t))$, wages $(w(t))$, inventories $(I(t))$ and strikes $(s(t))$ takes the form of a differentral game. The union chosses the level of industrial action to carry out at each particular point in time, whereas the firm chosses prices and employment. In so doing they take in account two dynamic constraints: the outcome of the bargaining process (i.e. The wage dynamics) and the stock accumulation dynamics.

This framework is able to produce a set of Euler equations that may constitute the basis of an empirical rational expectations model. Unfortunately given its complexity it is helpless if one prefers closed form solutions or even to obtain clear cut results. Having three policy instruments and two state variables produces a hardly tractable dynamic structure with a system of six differential equations.

Once we depart from the simplest case of symetric players, choosing one single instrument each, and being constrained by the dynamics of one single state variable, the analysis of solutions becom[illegible]

This paper is to some extent a simplifying example of what can be attempted in the framework outlined at the begining of this section. For this reason, and to be able to get explicit solutions some further assumptions are needed. The union's size is assumed to be exogenously determined mainly on the basis of ideological prefferences and the like. This assumption is not difficult to accept per se, nevertheless in our context it would mean that strikes could have not only a role in the wage negotiations but also a signalling role for the militancy or otherwise of the union. This point is not considered to keep things simple. Similarly the treatment of uncertainty is naive and no attempt is made of capturing all the possible ways in wich uncertainty can be brougth into.

Section II presents the general dynamic structure of the oligopoly model, the saddle point nature of the equilibrium and the dynamics following alternative exogenous shocks are discussed. In section III we introduce the non-cooperative approach under alternative informational assumptions. The steady state results for the cooperative and non-cooperative scenarios are presented. We stablish a ranking of the equilibrium values of the main variables (wages, strikes, employment) and the short and long run multipliers following exogenous shocks. We obtain a clear ranking not only among the cooperative and non-cooperative solutions, but also between the open-loop and closed-loop ones; this remarks the importance of the informational assumptions in dynamic games. Section IV considers the simple model in an uncertain environment about the bargaining power of the firm. Under some simplifying assumptions about the stochastic nature of the shocks hitting the economy, we produce again an unambiguous comparision with the results in section III wich is explained in terms of the underlying risk aversion of unions. In the final section we present some special remarks and conclussions.

little illuminating. Numerical simulation and partial results are then required.To avoid this and to obtain clear-cut results we develop some simpler models, focusing on one or two main features at time.

In this paper I put aside the main purpose of the research, to put forward a rather simpler model in wich some of the features of what is coming next, can be shown more clearly. Having two (or n for what matters) symetric players and a simple state variable permits us to carry out a simple analysis in a world of two dimensions. Proceeding in this way we can also take some already well stablished results in the field of international policy coordination (Miller and Salmon (1983)) or oligopolistic ressource markets (Van Der Ploeg (1984)).

Only one side of the ultimate problem is taken here. In this example two active unions choose the optimal level of industrial action, taking in account the wage dynamics constraint as well as the labour demand shedule. The firm here plays no role other than a passive follower choice of the optimal level of employment taking wages as given.

Athough the simplicity of the framework must prevent us from drawing too occurrate conclussions out of the analysis, the study of a world with several unions is both intuitively and in practice rather more appealing. In many european countries several unions exist in the same sectors stablishing wage contracts wich hold for any worker in the sector regardless of his union status. It can be interesting then to discuss a little bit what difference does it make wather unions cooperate or not, and in general the alternative informational assumptions wich are crucial in dynamic games.

Oswald (1979), has studied the wage determination in a model with several unions in an oligopolistic framework. our approach differs from his in two main aspects. Following the previons discussion the wage is no longer the union's policy instrument; it is only the wage growth what way be (partly) [illegible] furthermore, [illegible] is homogeneous and each union negotiates for all the workers.

II THE COOPERATIVE SOLUTION

Let us consider two unions and a single firm. Each union cares about the utility of its own members, but in so doing signs up contracts of overall applicability. The wage equation follows from the discussion in the previous section, and takes on the form.

$$\dot{w}(t) = -\beta w(t) + \alpha (S_1(t) + S_2(t)) + \gamma_w(t) \quad (3)$$

where the wage outcome depends negatively on the current wage and positively on the level of industrial action in the manner already discussed. γ_w captures here a whole bunch of exogenous factors wich may affect the wage outcome trough the bargaining power of each party. The objective function of the i_{tn} player is.

$$\underset{s_i(t)}{\text{Max}} \int_0^T U_i(t) \exp\{-rt\} \, dt \quad (4)$$

where

$$u_i(t) = w(t) \; n_i(t) - \tfrac{1}{2} S_i^2(t) \quad (5)$$

The utility is affected by the wage bill in a conventional manner minus the cost associated to industrial disputes. The labour demand schedule is finally the conventional one:

$$n(t) = \gamma_n(t) - \psi W(t) \quad (6)$$

γ_n is again a shift factor wich captures any (exogenous) element in the labour demand other than wages, i.e. productivity, demand, etc......
I finally assume full symetry and hence

$$n(t) = n_1(t) + n_2(t) = 2n_i(t) \quad (7)$$

notice finally that all $\beta, \alpha, \psi > 0$.

Let us analyze the cooperative solution (c) wich will be useful as a benchmark. By this we mean that the two unions get together to decide the optimal S_i. They do so by maximizing a weighted average of the two objective functions subject to all the constraints in the model. I assume an equal weight on each union's welfare function.

The solution is obtained by an straightforward application of the optimal control tecniques. Let the current value hamiltonian defined for T

$$H^c = H^c\,(W(t),\ S_1\,(t),\ S_2\,(t),\ \mu^c\,(t))$$

$$H^c = \tfrac{1}{2}\,(W(t)\ \gamma_n\,(t)\ - \psi W^2\,(t)) - \frac{1}{4}\,(S_1^{\,2}\,(t) + S_2^{\,2}(t)) +$$

$$\mu^c\,(t)\ \big(-\beta\, W\,(t) + \alpha\,(S_1\,(t) + S_2(t)\big) + \gamma_w\,(t)) \qquad (8)$$

which gives:

$$H_s = -\tfrac{1}{2}\, S_1 + \alpha\,\mu^c = 0 \qquad (9)$$

$$H_s = -\tfrac{1}{2} S_2 + \alpha\,\mu^c = 0 \qquad (10)$$

$$H_w = \tfrac{1}{2}\,(\delta_n - 2\psi W) - \beta\mu^c = -\dot{\mu}^c + r \qquad (11)$$

(9) and (10) are the usual first order conditions (F.O.C.) for an optimum. The interpretation is in the usual marginal sense; it is worth noting how (as it couldn't be otherwise) strikes are observed at all so long as the union has got a non zero bargaining power ($\alpha \neq 0$). The union will strike until the point in wich the marginal cost of further action ($\tfrac{1}{2}S_i$) equals the marginal valuation in terms of higher wage.

(11) is the arbitrage condition that resulting wages must satisfy at the optimum.

$$\frac{\dot{\mu}^c}{\mu^c} + \frac{\lambda}{2\mu^c}\left(\gamma_n - 2\psi w\right) = r + \beta \qquad (11')$$

LHS (11') captures the expected rate of return of yet a further small increase in wages, it consists of two elements. $\left(\dot{\mu}^c/\mu^c\right)$ is the capital gain and $\left(\frac{1}{2\mu^c}\left(\gamma_n - 2\psi w\right)\right)$ is the current return bit as the increase in the wage bill. RHS (11') is on the other hand the rate of return that can be obtained from the allocation of those ressources needed to increase W to an alternative use. r es the pure financial return that the ressources used to get higher wages (strike costs and so on...) could achieve somewhere else in a perfect capital market, and β is the return in terms of future wage increases.

The next step is to convert the set of (9) - (11) conditions into a simple system of differential equations, to study both the dynamics and the steady state (for $T \longrightarrow \infty$) of the system. Exploiting the symetry of the model we can aggregate over (9) and (10) ans use the resulting equation plus (11) and (3) to get (Appendix 1):

$$\begin{bmatrix} \dot{W} \\ \dot{S} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} W \\ S \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} \gamma_w \\ \gamma_n \end{bmatrix} \qquad (12)$$

Where:

$a_{11} = -\beta$

$a_{12} = \alpha$

$a_{21} = 4\alpha\psi$

$a_{22} = (\beta + r)$

$b_{11} = 1$

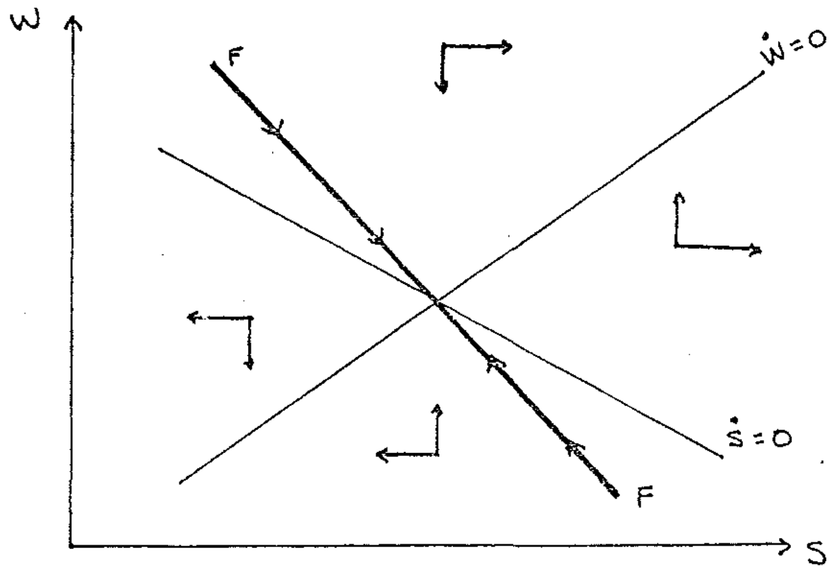
$b_{12} = 0$

$b_{22} = -2\alpha$

There are two things to care about in system (12), wether it satisfies the saddle point condition and how do the long run multipliers look like. Let the matrices $A^c = \{a^c_{ij}\}$ $B^c = \{b^c_{ij}\}$ and is easy to chech how the saddle point condition is satisfied for:

$$\Delta^c = \det A^c < 0$$

Diagramatically is easy to check how system cam be represented by the foollowing phase diagram



[Fig. 1]

Furthermore, the steady state values of W_c, S_c are given by

$$W_c = -\frac{1}{\Delta^c}\left[\gamma^*_w(\beta+r) + 2\alpha^2\gamma^*_n\right] \tag{13}$$

$$S_c = -\frac{1}{\Delta^c}\left[2\alpha\left(\beta\gamma^*_n - 2\psi\gamma^*_w\right)\right] \tag{14}$$

I shall assume $\beta\gamma_n > 2\psi\gamma_w$ such that we get a meaningful steady state (W_c, S_c) in the positive cuadrant (NOTE 1) . γ_n^*, γ_w^* are the corresponding steady state values of the exogenous shift factors.

The steady state solutions for n and μ can also be shown for completeness.

$$n_c = -\frac{1}{\Delta^c}\left[\gamma_n^*\left(\beta(\beta+r)+2\alpha^2\psi\right)-\gamma_w^*\psi(\beta+r)\right] \tag{15}$$

which is also positive if the condition for positiveness of S_c is met.

Finally

$$\mu_c = -\frac{1}{\Delta^c}\left[\frac{1}{2}\left(\beta\gamma_n^*-2\psi\gamma_w^*\right)\right] \tag{16}$$

wich is also positive under the same assumption.

Notice how (16) shows the steady state value of μ as the present discounted value of the future stream of increases in wage bill due to an increase in wages. Actually (16) could be written as

$$\mu_c = \lim_{T'\to\infty}\int_T^{T'} \frac{1}{2}\left.\frac{\partial(wn)}{\partial w}\right|_{\substack{w_c\\ n_c}} \exp\{-(r+\beta)(\tau-T)\}\, d\tau \tag{16'}$$

For in the steady, when no further changes in the exogenous variables are expected, the marginal valuation of a further steady state wage increase is the PDV of all future wage bill increases discounted back at the effective rate $r + \beta$.

The long run multipliers are easily obtained from (13)-(15). Let's show them and introduce a few interpretative comments.

$$\frac{dW_c}{d\gamma_w} > 0 \quad (17) \qquad \frac{dW_c}{d\gamma_n} > 0 \quad (18)$$

$$\frac{dS_c}{d\gamma_w} < 0 \quad (19) \qquad \frac{dS_c}{d\gamma_n} > 0 \quad (20)$$

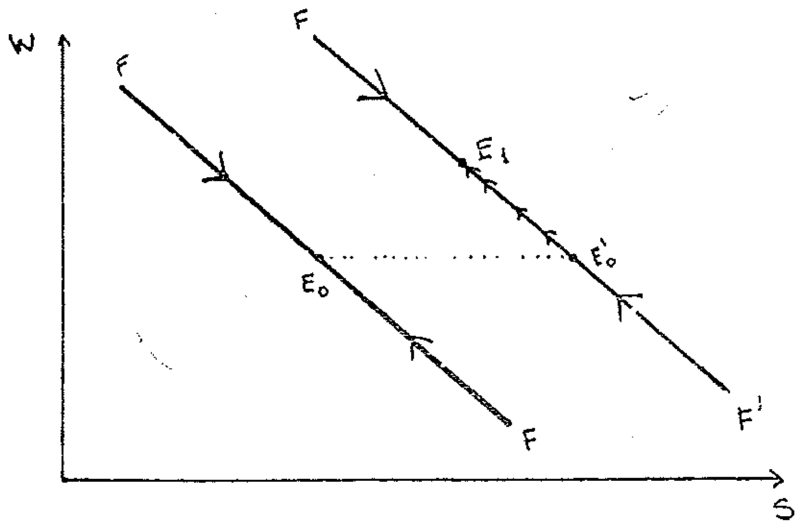
$$\frac{dn_c}{d\gamma_w} < 0 \quad (21) \qquad \frac{dn_c}{d\gamma_n} > 0 \quad (22)$$

$$\frac{d\mu_c}{d\gamma_w} < 0 \quad (23) \qquad \frac{d\mu_c}{d\gamma_n} > 0 \quad (24)$$

(17) and (18) are fairly clear. Positive shifts in γ_w, $\dot{\gamma}_n$ improve worker's bargaining power and hence lead to higher wage growth. (19) reflects how this improvement in the bargaining power permits a reduction in the number of strikes to achieve a real wage target. (20) however shows how a productivity shock (say) wich does not produce an inmediate wage push but a higher labour demand, must be gained by workers as higher wages through an increasing industrial action. (21) (22) show the unequal impact on employment of two wage increasing shocks. (23) (24) tell us how a pure wage shock leads to a higher wage but reduces the marginal valuation of further increases; when the shock is employment increasing, though, it leads to a higher marginal value of the state variable.

It is worth spending some time in analysing carefully the short run dynamics of the system for it will become an interesting point for comparision when we get to another forms of solution.

A positive productivity shock leads the system from E_o to E_1 wich lies unambiguosly to the north east of E_o. It is easy to show how the slope of EE is negative and hence strikes overshoot following an increase in γ_n (Appendix 2).



[Fig. 2]

Let the economy at the steady state Eo being hit by and unanticipated permanent increase in productivity at to.

The economics behind it seem a bit obscure as it is difficult to think why rational unions should increase S on impact precisely from $S(E_o)$ to $S(E'_o)$, and then go along the new stable manyfold towards $S(E_1)$. The reason can be better understood by looking at equations (1.1) (1.2) in Appendix 1.

Union's optimal choice of S depends on its valuation of further wage increases, μ. When the labour demand goes up because of an increase in γ_n the valuation of higher wages goes up and then it smoothly settles at new steady state level μ (E_1). The explanation of why S overshoots relies here on the explanation of the behaviour of μ, for

$$ds = 4\alpha \, d\mu \qquad t = t_o^+ \tag{25}$$

$$\dot{S} = 4\alpha\dot{\mu} \qquad t_o^+ < t < \infty \tag{25'}$$

when γ_n increases so does n and w doesn't change for it is a predetermined variable. These two reasons work to make higher wages more desirable and hence μ wich is a non predetermined shadow price goes up accordingly. But it does more than that, and it overshoots its long run level. Rational agents know that wage (employment) will be hihger (lower) on in the long run (as $t \to \infty$) than it is at t_o. So they value further wage increases at t_o, when the increase hasn't taken place yet, more than they will at $t \to \infty$. μ must then overshoot.

Rational unions look only at their valuation μ as to decide their strike policy. This leads to a sudden increase in industrial action wich will die away smoothly as the objective of higher wages is apporoached and the cost in terms of falling employment becomes heavier.

Notice that we are talking about unanticipated permanent shifts in γ_n, and this leads us to the second point wich is worth noting. After the initial jump, when the new saddle path is approached, the economy goes along the new saddle path F'F'. It is shown in the Appendix 2 how the dynamics along the stable many fold is given by

$$\dot{W} = \lambda (W - W_c) \tag{26}$$

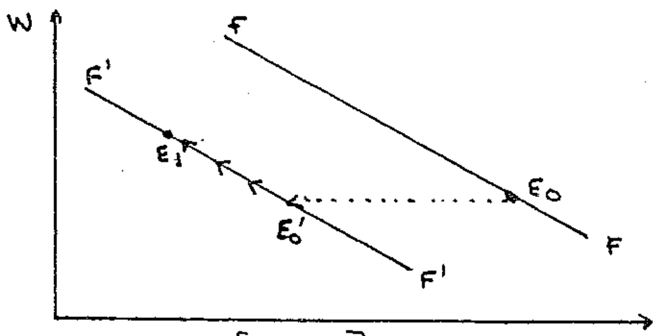
$$\dot{S} = \lambda (S - S_c) \tag{27}$$

and consequently

$$\dot{n} = -\lambda \psi (w - w_c) = \lambda (n - n_c) \tag{28}$$

where λ is the stable (negative) eigenvalue of Δ_c. It turns out to be the case that in this case the dynamics of the endogenous variables is similar to the partial adjustment mechanism sometimes used for empirical purposes. However this only holds for unanticipated permanent shocks hitting the system, i.e. random walk like. Otherwise the partial adjustment mechanism doesn't follow from a rational behaviour. To put it in other terms, the parameter of the adjustment is not structural and partial adjustment models are Lucas Criticable.

A positive wage shock moves the economy from E_0 to E_1 in the long run. And is easy to show that E_1, must lie northwestwards with the new stable locus (F'F') going below the old one (FF), as in (Fig. 3).



[Fig. 3]

Unlike the case above, if the shock is unanticipated and permanent, the shadow price of w undershoots its long run fall. On impact this reduction goes even further in the long run when the increase in wage lowers employment. So μ goes on falling until a new long run equilibrium is achieved. The path of S(t) is traced by that of μ (t), in the manner discussed above.

Notice how this result makes sense because what a positive wage shock means here is an increase in the bargaining power of the union. This produces on immediate drop in the strikes actually taking place; but as time goes by and wages are actually increased there is a further discouraging effect on strikes due to a favourable position in the state variable. This asymetric behaviour of wages and strikes following different shocks is interesting and consistent with some well stablished results in labour economics. The shape of the wage dynamics constraint apparently reflects a causal relationship running from strikes to wages such that the higher the former the higher the wage setlement achieved. This is not the whole story ; in our model both strikes and wages are endogenously determined and hence the nature of the dynamic relationship among them depends on the type of the exogenous shocks. Finding a positive assotiation between strikes and wages in some cases, is not inconsistent with the claim that strong unions (in the sense of unions having a high bargaining power or high γ_w) do need less industrial action to achieve a given pay target. This issue is analyzed in a somehow different context by Gerosky et al (1984) (1985) and their results support to some extent this complex relationship among wages and strikes.

Let us finally get an expression for the discrete initial jump in S(t) following an unanticipated permanent change in either γ_n^* or γ_ω. Recalling the equation for the stable locus in eq. (2.5) of Appendix 2, is easy to check how the horizontal (impact) change in S is given by, (NOTE 2)

$$dS(t) = \frac{\lambda}{\alpha \Delta^c}\left[2\alpha^2 \, d\gamma_n^*\right] > 0 \qquad t = t_0^+ \qquad (29)$$

$$dS(t) = \frac{\lambda}{\alpha \Delta^c}\left[(\beta + r) - V\right] d\gamma_\omega^* < 0 \qquad t = t_0^+ \qquad (30)$$

for $d\gamma_\omega^* d\gamma_n^* > 0$ where V is the unstable (positive) eigenvalue of Δ^c.

III. A NON-COOPERATIVE APPROACH

3.1.-NASH OPEN LOOP SOLUTION

The cooperative solution (c) is a useful benchmark with wich is posible to compare the non-cooperative ones. The open-loop Nash assumption (OL) is the most natural alternative in this case. As was pointed out in the previous sections this solution implies that each agent is solving an optimal control problem independent of the other's.

To make this feasible the Nash assumption, i.e.

$$E_2\left(\frac{\partial S_1}{\partial S_2}\right) = E_1\left(\frac{\partial S_2}{\partial S_1}\right) = 0$$

is not enough. We actually recquire this sort of myopic behaviour in wich each agent expect the other's instrument not reacting to changes on its own, to prevent from any form of leadership (NOTE 3). But we also consider static information pattern so that no feedback is expected from the current state to the instrument. In particular (and now the time subscript becomes relevant).

$$E_2\left(\frac{\partial S_1(t)}{\partial w(t)}\right) = E_1\left(\frac{\partial S_2(t)}{\partial w(t)}\right) = 0$$

Is under these two assumptions that we can state the i_{tn} player optimal control problem as:

$$H_i = \frac{1}{2}\left(w(t)\gamma_n(t) - \psi\, w^2(t)\right) - \frac{1}{2}\left(S_i(t)\right)^2 + \mu_i(t)\left(-\beta w(t) + \alpha\left(S_1(t) + S_2(t)\right) + \gamma_w(t)\right) \qquad (31)$$

wich yields.

$$H_{i_{S_i}} = -S_i + \alpha\mu_i = 0 \qquad (32)$$

$$H_{i_w} = \frac{1}{2}\gamma_n - \psi w - \beta\mu_i = -\dot{\mu}_i + r\mu_i \tag{33}$$

and upon aggregation accross players, where we assume

$$S_1 + S_2 = 2S_1 = 2S_2 = S$$
$$\mu_1 + \mu_2 = 2\mu_1 = 2\mu_2 = \mu$$

we get

$$S = \alpha\mu \tag{34}$$

$$\dot{\mu} = (\beta + r)\mu - (\gamma_n - 2\psi w) \tag{35}$$

and proceeding as we did in the (C) case we end up with

$$\begin{bmatrix} \dot{w} \\ \dot{s} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} w \\ s \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} \gamma_w \\ \gamma_n \end{bmatrix} \tag{36}$$

where
$a_{11} = -\beta$
$a_{12} = \alpha$
$a_{21} = 2\alpha\psi$
$a_{22} = (\beta + r)$
$b_{11} = 1$
$b_{12} = 0$
$b_{21} = 0$
$b_{22} = -\alpha$

Is easy to check that the system satisfies the saddle point property condition, because

$$\Delta_{OL} = \det A_{OL} < 0$$

where $A_{OL} = \{ a_{ij}^{OL} \}$, $B_{ON} = \{ b_{ij}^{OL} \}$

All the dynamics we analyzed for the case of cooperation (C) still go through. It is easy to check how the phase diagram shows a do..ard slopping stable manyfold as in the cooperative case. The short run and long run dynamics go then in the same direc...n as they did in that setting. It is no worth going again into them.

What is more interesting though, is to compare the "size" of these effects. Given the extremely simplifing structure of the model, it turns out to be the case that most of these comparisions can be made just looking at the relative size of the relevant dete..minants, i.e. Δ_C vs. Δ_{OL}

Let us see how the steady state solutions look like.

$$w_{OL} = -\frac{1}{\Delta_{OL}}\left[\gamma_w^* (\beta + r) + \alpha^2 \gamma_n^* \right] \tag{37}$$

$$s_{OL} = -\frac{1}{\Delta_{OL}}\left[\alpha \left(\beta \gamma^* - 2\psi \gamma_w^* \right) \right] \tag{38}$$

$$\mu_{OL} = -\frac{1}{\Delta_{OL}}\left[\left(\beta \gamma_n^* - 2\psi \gamma_w^* \right) \right] \tag{39}$$

$$n_{OL} = -\frac{1}{\Delta_{OL}}\left[\gamma_n^*\left(\alpha^2\psi + \beta(r+\beta)\right) - \gamma_w^*(\beta+r)\psi\right] \tag{40}$$

under the assumption we made before

$$\beta\gamma_n^* > 2\psi\gamma_w^*$$

all are positive.

Comparison among (24) and (56) shows how.

$$S_c > S_{OL} \qquad \text{(NOTE 8)}$$

and similarly from (13) (15) (16) (37) (39) (40):

$$W_c > W_{OL}$$

$$n_c < n_{OL}$$

$$\mu_c < \tfrac{1}{2}\mu_{OL}$$

What is worth explaining is why unions strike more in a cooperative than in a non cooperative solution. The results for wage and employment follow from the different use of the policy instrument in each case.

If we look at (16) and (39), we can see how the shadow price of further wage increases is higher in the non cooperative case. Actually $\frac{1}{2}\mu_c$ is the valuation for each player of yet another increase in wages if in the non-cooperative case, and μ_c is the corresponding value that the union cartel places on such increase. The different behaviour is, obviously, explained by the sort of cooperation among the unions. In the non-cooperative case, each union increases the industrial action up to the level in wich the cost of further strikes increases by more than the corresponding return. But each union does not expect the other instrument's being affected by what it does, so the expected marginal return of further striking is given by

$$S_i^{OL} = \frac{1}{2} \alpha \mu_{OL} \tag{41}$$

The underlying behaviour in the cooperative case is the same. Now, however, each union things that the other is going to increase the number of strikes exactly by the same amount than it does. The conjectural variation is then equal to one. The marginal return for the union of an increase in strikes is the sum of its effect upon wages plus the effect of the corresponding increase in the other union's strikes.

$$S_i^{c} = 2 \alpha \mu_c \tag{42}$$

Therefere, altough $\mu_c < \frac{1}{2} \mu_{OL}$, is easy to show how $2\mu_c > \frac{1}{2}\mu_{OL}$ and hence each union leads the amount of strikes to a higher level under cooperation (NOTE 4).

The (OL) solution does satisfy the saddle point equilibrium condition too. The short run dynamics following a given shock will be similar to the cooperative case. However the size of both the long run and the very short run (impact) changes are likely to be different. Two results come up from the steady state analysis, i.e. refering to the long term impacts.

If a positive pure wage shock hits the economy $(\Delta \gamma_w^* > 0)$, the increase in wages is smaller and the fall in strikes bigger under a cooperative behaviour than in a non cooperative one. When a positive employment shock comes, the results are quite different; both wages and strikes rise by more if the unions do cooperate.

$$\frac{dw_c}{d\gamma_w^*} < \frac{dw_{OL}}{d\gamma_w^*}$$

$$\left|\frac{ds_c}{d\gamma_w^*}\right| > \left|\frac{ds_{OL}}{d\gamma_w^*}\right|$$

$$\frac{dw_c}{d\gamma_n^*} > \frac{dw_{OL}}{d\gamma_n^*}$$

$$\frac{ds_c}{d\gamma_n^*} > \frac{ds_{OL}}{d\gamma_n^*}$$

The results for employment follow inmediately from what happens to wages.

To understand such asymetric response to different shocks, it is, as always, worth looking at what do these shocks do to the shadow price of wages. This is inmediately transmited to the union reaction in terms of strikes and that explains the long run impact on wages. The main difference among a pure wage and an employment shock lies on their impact upon μ.

A pure wage shock relaxes the wage constraint by producing an inmediate increase in the rate of growth of wages. This sudden drop in μ is what causes a reduction in the number of strikes. But the impact upon strikes is bigger under cooperation. Under (OL) each union only cares about its own valuation of wages, hence a drop on it causes a moderate reduction in the amount of strikes. Under (C) though, each union is striking higher because it takes in account the return (in terms of higher wages) arising to both unions, so when μ falls the observed return falls by more than under non-cooperation leading to a bigger drop in strikes and so to a smaller increase in wages.

The same sort of reasons explain the bigger response of both wages and strikes to a productivity shock under cooperation. This is, in a way, much more illustrative of what the effect of union cooperation is about, for employment shocks are more frequent and realistic. A productivity shock does not relax the dynamic constraint, but, on the contrary, by rising employment makes the shadow price of wage increases to increase further. The incentive for the unions to capture part of this productivity shock is higher given its impact upon employment. By the same reasons as before, this increase in μ leads to a bigger rise in the marginal return of further strikes under (C) than under (OL). As a consequence of that, cooperant unions are able to get a higher share of productivity growth in terms of higher wages, leading to a worse impact in terms of employment.

Let us finally comment slightly on the impact effect wich can be more interesting as far as the cycle is concerned. As we saw earlier, and for the saddle point property, the number of strikes jumped on impact following an unexpected shock. It is interesting to see apart from the long run changes, wether the sort run impact of an employment shock (say) does differ wether we are under cooperation or not. To see this, we must calculate the horizontal shift of the stable manyfold, for it defines, as we saw, the of the size jump in the forward looking variable, following an unexpected shock wich is seen as a permanent one. The figure 4 shows the jump we try to measure:

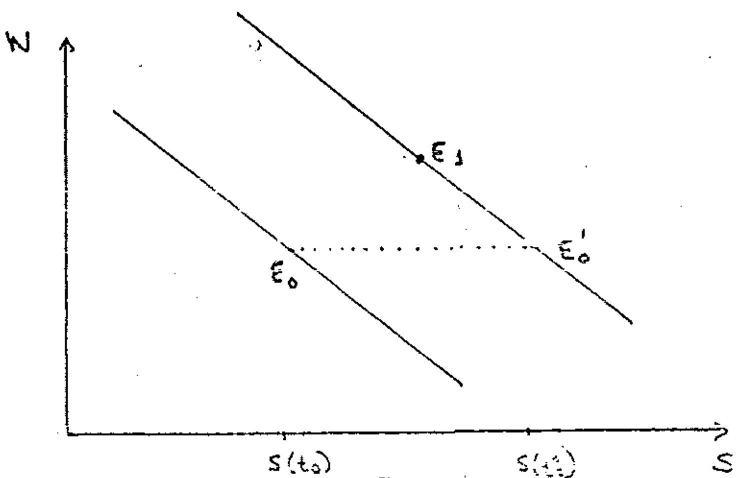


Fig.4

From (29) we know that the impact effect upon $S(t_o)$ following a shock at to is given by

$$dS_c^c(t) = \frac{2\lambda_c}{\Delta_c} d\gamma_n^* \qquad t = t_o^+$$

under (C). It is easy to how the corresponding effect under (OL) is given by

$$dS_{OL}(t) = \frac{\lambda_{OL}}{\Delta_{OL}} d\gamma_n^* \qquad t = t_o^+$$

Then the impact effect following a similar pattern to the long run ones (NOTE 5)

$$\left.\frac{dS_{OL}}{d\gamma_n^*}\right|_{t=t_o^+} < \left.\frac{dS_c}{d\gamma_n^*}\right|_{t=t_o^+}$$

If unions cooperate they are not only able to get a larger share of positive employment shocks than under non-cooperation; they also do get it more quickly and through a bigger increase in the strike activity over the long run. Strategic behaviour means that every union expects a little return to its effort and so it does not get into highly active industrial action. If they do cooperate though the same cost lead to higher returns hence strikes must rise above the non cooperative level. This is also translated into the short run. More turbulence is then expected under well organiced unionism as well as a more efficient outcome of it.

3.2.- NASH CLOSED LOOP SOLUTION

Closed loop solutions are associated to dynamic information patterns. As time goes by, people's information sets get enlarged by the new coming information. Policy rules can then be made dependents upon current and past values of the state variables. This means that each agent expects the other to react to the evolution of the wage,and this effect is taken in account when doing maximization. In particular now:

$$E_i\left(\frac{\partial S_j(t)}{\partial w(t)}\right) \neq 0$$

The optimal plan for the i_{th} union now does allow for the expected impact of changes in current wages upon the other union's strike activity. The derivation of the optimal behaviour under closed loop is an straightforward exercise that can be carried out using the maximun principle on (41). The results obtained only differ from those under OL, in the shadow price equation wich now takes in account the new arising information in determining the co-state dynamics. Closed loop solutions present however conjectural non uniqueness (See Basar & Oldser (1982) for a discussion of this issue). What this means in essence is that under a dynamic information set, a given strategy admits infinite representations (i.e. it can be written in a number of ways as different functions of different elements in the information set) and there exist as many Nash equilibria as different representations of the optimal strategies of each agent we could think of.

In a deterministic context there is only a way of avoiding this problem. It consists of restricting our attention to a particular representation of each strategy; this means impossing the additional constraint for the solution being a function of a restricted set of elements in the information set. This permits us to pin down a single "representative" of each trajectory and hence of the optimal ones, restoring then the uniqueness of the Nash solution.

Among those restricted set of solutions there is one wich has got particular appealing, this is the so called Feedback solution (F). These strategies are the control functions S^*_{it} wich satisfy:

$$S^*_{it} = S_i(W_t) \tag{43}$$

and S^*_{it} satisfy the Optimality Principle, i.e. there exist functions J(t,W) such that

$$J^N(t,W) = \max \left[\int_t^{t+\Delta t} u(w,n,s^*) \exp\{-r\tau\} d\tau + J(t+\Delta t, W+\Delta W) \right] \tag{44}$$

What is meant by (43) is that the optimal (F) strategies only depend on the current value of the state. Each agent is assumed to be in possesion of a Feedback information set. This helps to get a unique representation of the strategy as was intented; but the (F) solution posses also another feature wich makes it prefferable to any other unique representation. It must satisfy the Bellaman's Principle of Optimality, i.e. it is an optimal trajectory given the inherited state and the expected optimizing behaviour. If the union was allowed to reoptimice at any moment it wouldn't choose a different strategy from S*. It is then subgame perfect wich makes it most desirable. The solution is relatively easy to obtain by an straightforward use of the Principle of Optimality:

Let us define

$$V^{N1}(t,w) = \max_{S_1^*} \int_t^T u_1 \exp\{\tau - t\} d\tau \tag{45}$$

where $V^1(t,w)$ is the present value maximun value function of union 1 corresponding to the optimal choice of its instrument S_1. The application of the Bellman's Principle permits us to writte.

$$-V_t^{N1} = \max_{S_1^*} \left\{ u_1 e^{-rt} + V_w^{N1} \frac{dw}{dt} \right\} \tag{45'}$$

and by defining thc current maximun value function J'

$$V^{N1}(W) = e^{-rt} \quad J^{N1} \tag{46}$$

So that

$$V^{N1} = e^{-rt} J^{N1} \tag{46'}$$

$$\dot{V}^{N1} = e^{-rt} \dot{J}^{N1} - re^{-rt} J^{N1} \tag{46''}$$

that we redefine (45') as

$$rJ^{N1} - J_t^{N1} = \max_{S_1} \left\{ u_1 + J_w^{N1} \frac{dw}{dt} \right\} \tag{47}$$

The optimality conditions are obtained from (47) in the usual manner. Differentiating w.r.t. s_1, we get the optimal feedback plan for $S_1(t)$, as

$$-S_1 + \alpha J_W^{N1} = 0 \qquad (48)$$

and the arbitrage condition is obtained upon differentiating (47) w.r.t W(t), and applying the total differentiation rule

$$r J_w^{N1} - J_{tw}^{N1} = \frac{1}{2}(\gamma_n - 2\psi w) + J_{ww}^{N1}\dot{w} + \alpha J_w^{N1}\frac{\partial S_2}{\partial w} - \beta J_w^{N1}$$

or

$$\dot{J}_w^{N1} = (r+\beta) J_w^{N1} - \frac{1}{2}(\gamma_n - 2\psi w) - \alpha J_{ww}^{N1}\frac{\partial S_2}{\partial w} \qquad (49)$$

where $$\dot{J}_w^{N1} = \frac{d J_w^{N1}}{dt} = J_{wt}^{N1} + J_{ww}^{N1}\frac{dw}{dt}$$

(49) is better interpreted on the light of (33). If we give to J_w^N the usual interpretation of the shadow price of W, (Kamien and Schwartz (1982)), (48) looks exacltly as (33) except for two bits. First of all, J_w^N does depend only ow current wages, whereas under open loop, the shadow price will depend on W(0). This is due to the dynamic programing nature of the (F) solution as well as to the information set. This ensures the condition of S* depending on current wages but not on past wage realizations.

More relevant indeed is the conjectural element $\partial S_2/\partial w(t)$. The reason why this appears in the arbitrage condition is due to the closed loop nature of the (F) solution. Each agent still expects no direct response of the others instruments to changes on its own $\left(\text{i.e. } E_j\, \partial S_i/\partial S_j = 0\right)$. However because optimal rules are based upon W(t) the optimizing union cannot ignore the other union strikes' response to changes in the wage level. This conjectural element is going to drive the difference between the (OL) and (F) solutions. It is interesting because it emphasizes the crucial role of the informational assumption in differential games as compared as with simple agent control theory.

The way in wich this is going to affect the optimal level of strikes and wages is clear from (48) (49). (48) is the same optimality condition as in (OL) except for its feedback nature, in the sense of giving as a result, an optimal strike rule as function of W(t). Where the conjecture about the other's response is going to be important, is in the arbitrage condition (49) to determine the expected return for the union of puting yet a bit more of pressure for higher wages.

$$\frac{\dot{J}_W^{N1}}{J_W^{N1}} + \frac{1}{2J_W^{N1}}\left(\gamma_n - 2\psi w\right) + \alpha L_2^1 = (r+\beta) \qquad (49')$$

where L_j^i represents the i_{tn} conjecture of j_{th} reaction to the state variable.

As it stands, RHS (49') is still the alternative rate of return of ressources devoted to obtain higher wages. LHS (49') is obviously the expected return of the last unit of ressources that the union spends (i.e. cost of strikes) in getting higher wages. (49') looks farly similar to the arbitrage conditions we saw earlier but it is augmented by L_2^1. Suppose L_2^1 is negative, it means that the first union expects a reduction in the industrial action to be taken by the other one as a consequence of higher wages; this obviously enters negatively in the expected rate of return of further strikes.

Let us now proceded to put the model in the same format as in the (C) and (OL) case. The state space representation is again of the form:

$$\begin{bmatrix} \dot{w} \\ \dot{s} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} w \\ s \end{bmatrix} + \begin{bmatrix} b_{11} & b_{21} \\ b_{12} & b_{22} \end{bmatrix} \begin{bmatrix} \gamma_w \\ \gamma_n \end{bmatrix}$$

where the matrices $A^F = \{a^F_{ij}\}$, $B^F = \{b^F_{ij}\}$ are given by:

$a^F_{11} = -\beta$
$a^F_{12} = \alpha$
$a^F_{21} = 2\alpha\psi$
$a^F_{22} = (r+\beta - \frac{1}{2}\alpha L)$
$b^F_{11} = 1$
$b^F_{12} = 0$
$b^F_{21} = 0$
$b^F_{22} = -\alpha$

where the steps to get (50) are the usual ones, i.e. aggregating over the FOC exploiting the symetry of the model:

$$S_1 = S_2 = \tfrac{1}{2}S, \; J^1 = J^2 = \tfrac{1}{2}J, \; L^1 = L^2 = \tfrac{1}{2} L$$

The interesting comparison among open-loop and feedback non cooperative solutions only depends on the sign of L, for the models are otherwise equal. In particular let us show how the sign of L is bound to be negative. If we take the first order condition (48) we can see how the marginal condition is affected by a change in W. In particular

$$\frac{\partial S_1}{\partial w} = \alpha \frac{\partial J^1_w}{\partial w} < 0$$

The negative value of J^1_{ww} comes from the linear quadratic structure of the model. It can be shown (Malliaris & Brock (1982)) that it leads to a concave maximum value function on W. If that is the case, and the conjectures are to be consistent, it means that,

$$L_2^1 = \alpha J_{ww}^2 < 0 \tag{51}$$

$$L_1^2 = \alpha J_{ww}^1 < 0 \tag{52}$$

The impact of this on the long run and short run dynamics of the model is fairly obvious. First of all, it does mean that the system in (50) satisfies the saddle point equilibrium condition.

$$\Delta^F = \det A^F < 0$$

This is good enough, for it leads to the same shape of reponse that we have found so far and permits us to discuss what difference the information assumption makes, as far as the size of the short run and long run response of the system to exogenous shocks is concerned.

If we get the long run solutions of the model.

$$w_F = -\frac{1}{\Delta^F}\left\{(\beta + r - \tfrac{1}{2}L)\gamma_w^* + \alpha^2\gamma_n^*\right\} \tag{53}$$

$$S_F = -\frac{1}{\Delta^F}\left\{\alpha\left(\beta\gamma_n^* - 2\psi\gamma_w^*\right)\right\} \tag{54}$$

$$n_F = -\frac{1}{\Delta^F}\left\{\left[\beta(\beta + r - \tfrac{1}{2}\alpha L) + \alpha^2\psi\right]\gamma_n^* - \psi(\beta + r - \tfrac{1}{2}\alpha L)\gamma_w^*\right\} \tag{55}$$

$$J_{ww} = -\frac{1}{\Delta^F}\left\{\beta\gamma_n^* - 2\psi\gamma_n^*\right\} \tag{56}$$

It is easy to see how under our familiar assumption

$$\beta \gamma_n^* > 2 \psi \gamma_w^*$$

$S_F \angle S_{OL}$ and hence the following relations also go through.

$$W_F \angle W_{OL}$$

$$n_F > n_{OL}$$

$$J_W \angle \mu_{OL}$$

The rationale of why this happens is easly obtained from (49'). The setady state value of the shadow price is given by

$$J_W^1 = \lim_{T \to \infty} \int_T^{T'} \frac{1}{2} \frac{\partial(w\,n)}{\partial w}\Big|_{n^F_{WF}} \exp\left\{-(r+\beta-\alpha L_2^1)(\tau-T)\right\} d\tau \qquad (57)$$

The shadow price at the steady state ($t=T$) is the present discounted value of all future wage bill increases for the union membership. As it stands, it looks very much like the shadow price under open-loop; the interesting thing to notice is the different duscount rate. Under open-loop, the subjective discount rate is ($r+\beta$), wich is the expected rate of return of alternative use of ressources. Under closed-loop though this subjective discount factor is enlarged by ($-\alpha L_2^1$) because withdrawling some ressources from industrial action permits to the union to earn, r somewhere also and ($\beta-\alpha L_2^1$) in terms of future wage increases, for lower wages today mean higher wage growth both trough the "target real wage" bit ($-\beta w$) and the higher level of the rival's level of strikes as given by αL_2^1.

As far as the long run multipliers is concerned it is possible to show, how the following ranking can be stablished:

$$\frac{dW_{OL}}{d\gamma_n^*} > \frac{dW_F}{d\gamma_n^*}$$

$$\frac{dS_{OL}}{d\gamma_n^*} > \frac{dS_F}{d\gamma_n^*}$$

$$\frac{dW_{OL}}{d\gamma_\omega^*} < \frac{dW_F}{d\gamma_\omega^*}$$

$$\left|\frac{dS_{OL}}{d\gamma_\omega^*}\right| > \left|\frac{dS_F}{d\gamma_\omega^*}\right| \qquad \text{(NOTE 6)}$$

and the results for employment follow inmediately from these. Again there is an interesting asymetry in the response as compared with the (OL) case. In both sort of shocks the (F) response of strikes is smaller in absolute value than the (OL) one. The reason is to be found in the informational structure of each solution.

Take the productivity shock $(\Delta\gamma_n^*)$. The unions now try to gain a share in such shock in terms of higher wages and hence strikes go up. However, under (F), each union expects the other to reduce its industrial action following the increases in wages; hence this introduces a discouraging effect in for each union to strike further and hence aggregate strikes will not grow by that much. This leads to an smaller gain in terms of pay rise.

Exactly the opposite happens when a wage shock hits the system ($\Delta\gamma_\omega^*$). Both unions have an incentive to bring the strike activity down. However, at the same time each union realizes that its rival is going to [illegible] level [illegible] strikes due to the wage increase. This makes the [illegible] to expect a much smaller wage increase than the positive shock imitially permits, therefore each union has got an incentive not to reduce the number of strikes as much as it did under (OL). The aggregate industrial action falls but less than under (OL), and hence unions get a higher share in the positive wage shock.

For the sake of completeness, let us compare the very short run behaviour or the system following a positive (unanticipated and permanent) productivity shock. Proceeding in the same way as we did in previous cases, the impact effect is masured by the strikes boost following such shock, and it can be calculated as the horizontal shift in the stable manifold. It can be shown how this takes the form of

$$dS_F(t) = \frac{\lambda_F}{\Delta_F}\, d\gamma_n^* \qquad t = t_0^+$$

for a shock coming at t_o.

It follows inmediately from this that:

$$\left.\frac{dS_F}{d\gamma_n^*}\right|_{t=t_0^+} < \left.\frac{dS_{OL}}{d\gamma_n^*}\right|_{t=t_0^+} \qquad \text{(NOTE 7)}$$

wich also tells us that the presence of a dynamic information makes the unions more cautious on their short run and long run responses to exogenous shocks.

3.3.- FURTHER COMMENTS ON THE SOLUTIONS

As we have seen there is an interesting pattern coming out from the analysis we have just carried out. There is, in particular, a well defined pattern, wich features are worthwhile to analyse in more detail. As a sort of summary ot the previous results we can compare the solutions as follows:

$$\begin{array}{ccccc} W_F & < & W_{OL} & < & W_C \\ S_F & < & S_{OL} & < & S_C \\ N_F & > & N_{OL} & > & N_C \end{array} \qquad (C.1)$$

$$\begin{array}{ccccc} \dfrac{dw_F}{d\sigma_n^*} & < & \dfrac{dw_{OL}}{d\sigma_n^*} & < & \dfrac{dw_C}{d\sigma_n^*} \\ \dfrac{ds_F}{d\sigma_n^*} & < & \dfrac{ds_{OL}}{d\sigma_n^*} & < & \dfrac{ds_C}{d\sigma_n^*} \\ \dfrac{dn_F}{d\sigma_n^*} & > & \dfrac{dn_{OL}}{d\sigma_n^*} & > & \dfrac{dn_C}{d\sigma_n^*} \end{array} \qquad (C.2)$$

$$\begin{array}{ccccc} \dfrac{dw_F}{d\sigma_\omega^*} & > & \dfrac{dw_{OL}}{d\sigma_\omega^*} & > & \dfrac{dw_C}{d\sigma_\omega^*} \\ \left|\dfrac{ds_F}{d\sigma_\omega^*}\right| & < & \left|\dfrac{ds_{OL}}{d\sigma_\omega^*}\right| & < & \left|\dfrac{ds_C}{d\sigma_\omega^*}\right| \\ \left|\dfrac{dn_F}{d\sigma_\omega^*}\right| & > & \left|\dfrac{dn_{OL}}{d\sigma_\omega^*}\right| & > & \left|\dfrac{dn_C}{d\sigma_\omega^*}\right| \end{array} \qquad (C.3)$$

The main feature from (C.1)-(C.3) is the role played by the (OL) solution among the two other ones. This seems susprising at first glance, for (OL) seems to represent a total lack of cooperation, and information about the rival's response; whereas the F solution incorporates some (partial) view of what the other union is going to do. This again remarks the difference between cooperation and informational assumptions in this dynamic game.

Under (OL), both the Nash and the informational conjectures are set to zero. Each agent's expectations about its rival reactions are flat. Under (C) the increase in S_i is always simultaneous to an equal increase in S_j, hence the cartel takes in account such intereaction and finds worthwhile to increase both union's level of industrial action. Recall that the optimization procedure for a cartel leads to the same result that would have been achieved under non-cooperation proveded each agent expected : $\frac{dS_j}{dS_i} = 1$

Hence the cooperative solution can be interpreted either as a centralized cartel or an agreement whereby each union chosses its instrument optimally on its own, under the assumption that the other follows it on a one for one basis(APPENDIX 3). This obviously makes each union more keen on increasing the strike activity due to the additional bonus of the simultaaneous increase in the rival's pressure. As we see from (C.1)-(C.3), the level of strikes is always set up at a higher level under (C) than otherwise. The same pattern is observed for the long run multipliers and, as was shown earlier, for the impact effects. Strikes always react by more (in any direction) under cooperation, due to the encouraging effect of the induced response of the other union.

As it stands, the informational assumption plays an entirely different role. Altough the information contents of the (F) solution is higher than that of (OL) and so it seems to be halfway between (OL) and (C), actually the nature of the information introduces a discouraging effect on strikes. Under Nash, each union still guess a flat reaction from the other, to its strikes, hence no further incentive to increase industrial pressure works through this channel. But on top of that, because the union (correctly) guess the negative feedback from wages to the other's policy instrument if finds less interesting to strike as much as it would have done otherwise. In fact, the reaction of strikes (in either direction is bound to be always smaller under feedback than under any other solution

IV.- THE SOLUTIONS UNDER UNCERTAINTY

So far me have relied upon subgame perfect strategies to get rid of non uniqueness problems. There is however another natural way in wich dynamic programing can be justified as a way of solving the optimization program possed to the agentes.

When an stochastic element is brought into the system and under certain conditions of statistical independence, each strategy under closed loop information has got a unique representation. Therefore it is no longer possible to represent a given strategy in a different combination of past values of the state (Basar and Oldser (1982))because the realization of the disturbance drives a wedge between the expected and the actual dynamics of the system. Hence an optimal rule

$$S^*(t) = S^*(W(t))$$

cannot be written in terms of W(t-1) for the relationship between W(t) and W(t-1) includes a disturbance term, so that it is not possible to writte W(t) as a function of W(t-1) only.

The stochastic version of the model also permits us to draw some interesting conclussions about the way in wich true stochastic disturbances affect the behaviour of the agents. The modelling of uncertainty is deliberately naive, but it gives some insight of further ways in wich stochastic disturbances can be introduced.

I assume in particular that the relative bargaining power is now stochastic, and hence the wage increase is led by deterministic component, plus the unexpected shocks in the bargaining positions.

$$dw(t) = \left(-\beta w(t) + \alpha S(t) + \gamma_w(t)\right) dt + \sigma dZ \qquad (53)$$

dz is a Wiener process with zero mean and unit variance and σ is a function

$$\sigma = \sigma\left(w, S, \gamma_w, t\right) \tag{59}$$

wich tells us how a particular normalized shock affects the change in the state. The most important thing to bear in mind is that dZ are indpendently distributed with variance proportional to dt, and then equation (58) is no differentaible. Let us start by taking the simplest possible assumption about (59).

$$\sigma = \sigma(w) \qquad \sigma' > 0 \tag{59'}$$

4.1.- COOPERATIVE SOLUTION

If we define $\tilde{u} = \frac{1}{2} u^1 + \frac{1}{2} u^2$, the cooperative solution can be easily obtained. The optimal plan S* is the one wich satisfies the Hamilton-Jacobi-Bellman equation,

$$r J^c - J_t^c = \max_{S^*} \left\{ \tilde{u} + J_w^c g + \frac{1}{2} \sigma^2 J_{ww}^c \right\} \tag{60}$$

(APPENDIX 4)

For a well defined linear quadratic problem, there exists one function wich satisfies (60), hence we can procede straight to draw the main features of such solution.

The optimal plan is then obtained from (60) in the usual manner. The optimal level of strikes must maximize RHS (60)

$$-\frac{1}{2}s_i + J_W^c \frac{\partial g}{\partial s_i} = 0 \tag{61}$$

and the optimal wage level must satisfy the arbitrage condition

$$\begin{aligned} r J_W - J_{tW}^c &= \\ = \frac{\partial \tilde{u}}{\partial W} + J_W^c \left(\frac{\partial g}{\partial W}\right) &+ g J_{WW}^c + \sigma \sigma_w J_{WW}^c + \frac{\sigma}{2} J_{WWW}^c \end{aligned} \tag{62}$$

(61) and (62) can be rearranged in a familiar looking form

$$s_i = 2 \alpha J_{WW}^c \tag{61'}$$

$$\begin{aligned} \frac{1}{J_W^c}\frac{E dJ_W^c}{dt} + \frac{1}{2J_W^c}\left(\gamma_n - 2\psi W\right) &= \\ = -\sigma \sigma_W \frac{J_{WW}^c}{J_W^c} + r + \beta \end{aligned} \tag{62'}$$

where the Ito's rule has been applied to J_W

$$dJ_W^c = J_{Wt}^c\, dt + J_{WW}^c\, dW + \frac{1}{2} J_{WWW}^c (dW)^2 \tag{63}$$

$$E dJ_W^c = J_{Wt}^c\, dt + J_{WW}^c\, g\, dt + \frac{1}{2}\sigma^2 J_{WWW}^c\, dt \tag{64}$$

The interpretation of (61') is the one we saw in previous cases. So does (62') altough it contains a new bit wich captures a risk premium for the expected return on higher wages due to the uncertainty in the impact of future shocks. Let us analyze it with some more detail. (62') looks exactly as the arbitrage condition under certainty as given by (49'), except for the last bit. $\left(-\sigma\sigma_w J_{ww}/J_w\right)$ is positive under risk aversion (i.e. concave J), and it augments the alternative expected return. If the union does not "invest in higher wages" it does get (r+ β) as usual, plus the return of non increasing uncertainty. Actually, if w rises it increases its variance , and for a risk averse union this is utility reducing.

To analyze the consequences of introducing uncertainty we only compare feedback (cooperative vs. non cooperative) solutions as the only meaningful ones. In an stochastic world open loop strategies commit the players to a given behavioural rule with no account whatsoever for realized shocks. Fedback strategies on the other hand are much more realistics for they account for all information available up to t, and so they somehow react to past shocks. On the ohter hand the assumption of static information (open loop) structures is not a realistic one; its main role was in the deterministic setting to provide a benchmark for comparision, and furthmore to provide one solution for the non uniqueness problem. Now the problem of non uniqueness under closed loop information pattern is no longer present, so we can focus in a most interesting comparative analysis.

The concept of steady state equilibria we have been talking about so far, does not appear useful in an stochastic setting. There are several concepts of stochastic equilibrium (Malliardis and Brock (1982)), among the broader ones is that of stationary distribution. By this we mean, the existence of the solution of the stochastic differential equation wich is a time independent random variable. To get such a distribution for (58) we first need to writte it in the form:

$$dw(t) = h(w(t))\,dt + \sigma(w(t))\,dz \tag{65}$$

To do this we proced to substitute out S for tis optimal plan representation in terms of W(t' (the procedure is explained in some detail as it would be used later too).

From (61') and aggregating across S_1, S_2, we get

$$S = 4\alpha\ J^{c}_{W} \tag{66}$$

That can be expressed as

$$S = 4\alpha\left(K_1 - 2K_2 W\right) \tag{67}$$

where K_1 and K_2 are given by (VII.8) (VII.9) in the Appendix VII.

$$K_2 = \frac{1}{16\alpha^2}\left\{-(2\beta + r - \sigma^2) + \left[(2\beta + r - \sigma^2)^2 + 16\alpha^2\psi\right]^{1/2}\right\}$$

$$K_1 = \frac{1}{(\beta + r + 8\alpha^2 K_2)}\left(\frac{1}{2}\gamma_n - 2K_2\gamma_w\right)$$

Before to proceed further, a few comments are in order about the values of K_2 and K_1. It is easy to check how.

$$\frac{dK_2}{d\sigma} > 0 \qquad \frac{dK_1}{d\sigma} < 0$$

This is as one should expect, for increasing uncertainty must both, reduce the level of strikes for a given wage level and dampen the change in strikes in response to changes in wages. The uncertainty is associated here with higher wages and hence, unions will be more cautious in using the strike policy for the pay off of higher strikes is now lower. Similarly, any increase in the wage level will produce a bigger drop in the strike activity by the same reasons.

We can see how an increase in the elasticity of labour demand effects K_2, K_1 in the same way on this for obvious reasons. The higher the elasticity, the lower the pay off higher wages, so the use of strikes will be somehow reduced.

$$\frac{dK_2}{d\psi} > 0 \qquad \frac{dK_1}{d\psi} < 0$$

Now we can proceed to plugg (67) in (58) to get

$$dW\ (t) = (-\theta_1^c W(t) + \theta_2^c) + \sigma W\, dz \qquad (68)$$

where, $\theta_1^c = \beta + 8\alpha^2 K_2$

$\theta_2^c = \gamma_w + 4\alpha^2 K_1$

and if the stationary equilibrium distribution exists, it takes the form

$$\pi(w) = \frac{m}{\sigma^2 w^2} \exp\left\{2\int \frac{-\theta_1 w + \theta_2}{\sigma^2 w^2}\, dw\right\} \qquad (69)$$

or

$$\pi(w) = \frac{m}{\sigma^2} W^{2\left(\frac{-\theta_1 - \sigma^2}{\sigma^2}\right)} \exp\left\{-\frac{2\theta_2}{\sigma^2 w}\right\} \qquad (69')$$

where M, is such that

$$\int_0^\infty \pi(W)\ dW = 1 \quad \text{(Malliards and Brock(1982))}$$

The important thing about the existence of $\pi(W)$, is that it permits us an analysis close to the deterministic case. In particular it can be shown how,

$$E\left[\Gamma'(W)\ (-\theta_1^c W + \theta_2^c) + \tfrac{1}{2}\Gamma''(W)\ \sigma^2 W^2\right] = 0 \qquad (70)$$

and this is satisfied in our case for $\Gamma(W) = W$. (NOTE 8). Hence (70) becomes

$$E\left[-\theta_1^c W + \theta_2^c\right] = 0 \qquad (70')$$

or

$$E(W_c) = \frac{\theta_2^c}{\theta_1^c} \qquad (71)$$

And is precisely (71), what we can to compare how good is the aproximation of the deterministic steatd for the result under uncertainty. It can be shown that (13) can be written as:

$$W_c = \frac{\gamma_w + 4\alpha^2 \tilde{K}_1}{\beta + 8\alpha^2 \tilde{K}_2} \quad \text{(NOTE 9)} \tag{13'}$$

where

$$\tilde{K}_2 = K_2 \Big|_{\sigma=0} \qquad \tilde{K}_1 = K_1 \Big|_{\sigma=0}$$

This permits us to compare among the deterministic and the stochastic components, in similar grounds. Actually from (13') we have

$$W_c = \frac{\tilde{\theta}_2^c}{\tilde{\theta}_1^c} > \mathcal{E}(W_c) = \frac{\theta_2^c}{\theta_1^c}$$

and $$\frac{d\,(W_c - \mathcal{E}(W_c))}{d\sigma} > 0$$

As we saw earlier K_2 grows and K_1 falls with σ, hence for positive σ, the expected value of the stationary distribution of wages is lower than the deterministic steady state. By the same reason this difference gets bigger as uncertainty increases.

If we substitute out (71) into the expected value of (67) we can see how the expected stationary value of strikes is lower than the corresponding deterministic steady state value (NOTE 10).

4.2.-NON-COOPERATIVE SOLUTION

Bringing uncertainty in affects the non-cooperative solution in a similar manner than it did with the the cooperative one. Let us go quickly through the model. The optimal plan of each player must satisfy.

$$r J^{Ni} - J_t^{Ni} = \max_{S_i} \left\{ U^i + J_w^{Ni} g + \frac{1}{2}\sigma^2 w^2 J_{ww}^{Ni} \right\}$$

and hence

$$-S_i + \alpha J_w^{Ni} = 0 \tag{73}$$

$$rJ_w^{Ni} - J_{tw}^{Ni} = \frac{1}{2}(\gamma_n - 2\psi w) + J_{ww}^{Ni} g - \beta J_w^{Ni} + \alpha J_w^{Ni}\frac{\partial S_j}{\partial w} + \sigma^2 w J_{ww}^{Ni} + \frac{1}{2}\sigma^2 w^2 J_{www}^{Ni} \tag{74}$$

and using Ito's lemma we get, upon aggregation:

$$S = \alpha J_w^N \tag{75}$$

$$\frac{1}{J_w^N}\left(\frac{E_t dJ_w^N}{dt}\right) + \frac{1}{J_w^N}(\gamma_n - 2\psi w) = \left(\beta + r - \frac{1}{2}\alpha L\right) - \sigma^2 w \tag{76}$$

where, $S = S_1 + S_2$
$J = J^1 + J^2$
$L = L^1 + L^2$

The interpretation of (75) (76) is the usual one, except by the term ($-\alpha L$) wich is an augmenting bit in the alternative rate of return, due to the expected negative feedback from wages to the other union's strike level.

To be able to obtain an explicit solution for the expected value of the stationary distribution (the closest thing to the seterministic steadyd state concept), we proceed exactly as we did in the cooperative case. Let us writte.

$$S = \alpha\,(D_1 - 2D_2 w) \tag{77}$$

where,

$$D_2 = \frac{1}{5\alpha^2}\left\{(\sigma^2 - r - 2\beta) + \left[(\sigma^2 - r - 2\beta)^2 + 12\alpha\psi\right]^{1/2}\right\}$$

$$D_1 = \frac{1}{r+\beta+3\alpha^2 D_2}\left(\gamma_n - 2 D_2 \gamma_\omega\right) \quad \text{(NOTE 11)}$$

where again all the results for the case of cooperation, go through, so that we can writte the expected value of the stationary distribution under no-cooperation as:

$$E\ (W_N) = \frac{\theta_2^N}{\theta_1^N} \tag{78}$$

where

$$\theta_1^N = \beta + 2\alpha^2 D_2$$
$$\theta_2^N = \gamma_\omega + \alpha^2 D_1$$

wich is ooviously bigger than the steady state value in the deterministic case.

$$W_F = \frac{\gamma_\omega + \alpha_1^2 \tilde{D}_1}{\beta + 2\alpha^2 \tilde{D}_2} \tag{53'}$$

where

$$\tilde{D}_i = D_i \Big|_{\sigma=0} \qquad i = 1,2.$$

Here again the expected level of strikes and the expected (stationary) level of wages are both lower than the corresponding deterministic steady state values. The presence of uncertain randon shocks with affect the wage growth reduces the pay-off of strikes for risk averse unions.

Both results are obviosuly crucially dependent our the way in wich uncertainty has been broght into our model. Had the variance of the shocks been decreasing in wages we would have got exactly the opposite results.

V. CONCLUDING REMARKS

Under the simplifying assumptions put forward in the introduction, we have been able to derive closed form solutions for wages, strikes and employment, in a dynamic game model. The main results are the following ones.

The model displays a nice dynamic structure with the saddle point property. It enables us to pin down a unique dynamic path of wages, strikes and employment after en exogenous shock has taken place. When the bargaining power of the unions improves, wages go up whereas strikes and employment fall. A positive shock in the labour demand on the other hand increases wages, employment and strikes. We can find then different behaviour of the wage/strikes pair depending on the nature of the exogenous shocks in the economy.

Turning to the game theoretic approach we found a well stablished pattern of solutions between the cooperative and non-cooperative approach. These results are the ones one should expect and they may be taken as an straightforward extension of an static game between several unions. What is more interesting though, is the pattern we found under alternative informational assumptions in the dynamic context. In particular the difference between the cooperative and non-cooperative approach is systematically enlarged when a closed loop (feedback) information set is assumed for the unions. This pattern holds for both the short run and long run multipliers, as well as for the steady state solutions.

The introduction of uncertainty in the wage outcome makes the results persistently different from those above. In the last section we consider the bargaining power of the union subject to random shocks wich variance is increasing in the level of wages. In that case the expected value of the stationary distribution of wages and strikes is lower than the corresponding steady state solutions in the deterministic case. Employment is therefore greater in the uncertain case. This result is explained by the linear cuadratic format of the model wich makes the union risk averse and more keen in accepting lower wages (wich reduces the variance of the random shocks).

The natural extension of this model is to make explicit of the firm's bahaviour, endogenizing its role in the bargaining process. But before to do that it is also interesting making the sochastic nature of the model less naive. In particular the variance of the shocks affecting the relative bargaining power could be made dependent on the level of strikes as well. This could capture the fact that when a negative shock affects the union's bargaining power its impact upon the wage growth is bigger (worse) the higher the level of strikes. Industrial action would have not only a direct effect upon the wage outcome, but also an indirect one wich depends on the (favourable or otherwise) environment in wich nogotiations take place.

APENDIX 1

Aggregating ower (9) (10) in the text we get

$$S = 4\alpha\mu \tag{1.1}$$

so that

$$\dot{S} = 4\alpha\dot{\mu} \tag{1.2}$$

but from (11) in (1.2)

$$\dot{S} = 4\alpha\left[(r+\beta)\mu - \tfrac{1}{2}(\gamma_n - 2\psi w)\right] \tag{1.3}$$

but from (1.1) into (1.3)

$$\dot{S} = 4\alpha\left[(r+\beta)\frac{S}{4\alpha} - \tfrac{1}{2}(\gamma_n - 2\psi w)\right]$$

$$\dot{S} = 4\alpha\psi w + (r+\beta)S - 2\alpha\gamma_n$$

So that we have

$$\dot{W} = -\beta W + \alpha S + \gamma_w \tag{1.4}$$

$$\dot{S} = 4\alpha\psi W + (r+\beta)S - 2\alpha\gamma_n \tag{1.5}$$

APPENDIX 2

The solution of the differential system yields the following expressions

$$W(t) = C_1 x_1 e^{\lambda t} + C_2 x_2 e^{Vt} + W^* \tag{2.1}$$

$$S(t) = C_1 e^{\lambda t} + C_2 e^{Vt} + S^* \tag{2.2}$$

For a given steady state (W^*, S^*) the system is continnous in time. Furthermore, the transversality condition as we let $T \longrightarrow \infty$ is the condition of convergence towards the steady state. That recquires $C_2 = 0$ if we assume that the saddle point condition is satisfied and $V > 0$.

The system takes then the form of

$$w(t) = c_1 x_1 e^{\lambda t} + w^* \tag{2.1'}$$

$$S(t) = c_1 e^{\lambda t} + S^* \tag{2.2'}$$

where $\begin{bmatrix} x_1 \\ 1 \end{bmatrix}$ is the eigenvector associated to λ .

Differentiating with respect time we get (26) (27) in the text

$$\dot{W}(t) = \lambda (W(t) - W^*) \tag{2.3}$$

$$\dot{S}(t) = \lambda (S(t) - S^*) \tag{2.4}$$

wich only holds along the stable manyfold.

Given that λ is the stable (negative) eigenvalue its expresion is given by

$$\lambda = \tfrac{1}{2}\left\{(a_{11} + a_{22}) - \left[(a_{11} + a_{22})^2 - 4(a_{11}\, a_{22} - a_{21}\, a_{12})\right]^{1/2}\right\} \qquad (2.5)$$

and hence

$$\beta + \lambda = \tfrac{1}{2}\left\{(a_{22} - a_{11}) - \left[(a_{22} - a_{21})^2 + 4\, a_{21} a_{12}\right]^{1/2}\right\} < 0$$

hence the slope is negative, because if (2.3) and (2.4) are both equally meaningful representations of the stable manyfold we can proceed to get the slope as follows.

$$\dot{w} = \lambda (w - w^*)$$

hence $\dot{w} = -\beta w + \alpha s + \gamma_w = \lambda (w - w^*)$

or

$$w = \frac{\alpha}{\beta+\lambda} S + \frac{\lambda}{\beta+\lambda}(\lambda w^* + \gamma_w) \qquad (2.5)$$

is the equation for FF, and hence

$$\left.\frac{dw}{dS}\right|_{FF} = \frac{\alpha}{\beta+\lambda} < 0$$

APPENDIX 3

Imagine we let both unions to optimize on their own both under the conjectural assumption

$$E_i \frac{dS_j}{dS_i} = 1 \qquad \forall i,j=1,2$$

This would lead to maximiza

$$H_i^A = \frac{1}{2} wn - \frac{1}{2} S_1^2 + \mu_i^A \left(-\beta W + \alpha (S_1+S_2) + \gamma_w \right) \tag{3.1}$$

$$S_1 = \alpha \mu_i^A \tag{3.2}$$

$$\dot{\mu}_i^A = (r+\beta)\mu_i^A - \frac{1}{2}\left(\gamma_n - 2\psi w\right) \tag{3.3}$$

wich upon aggregation means

$$S = \alpha \mu^A \tag{3.4}$$

$$\dot{\mu}^A = (\beta + r)\mu^A - (\gamma_n - 2\psi w) \tag{3.5}$$

wich yields the same result as the cooperative case

$$S = 4\alpha\mu^c$$

$$\dot{\mu}^c = (r+\beta)\mu^c - (\gamma_n - 2\psi w)$$

$$\mu^A = 2\mu^c, \qquad \mu_i^A = \mu^c$$

APPENDIX 4

Let the (present value) maximum value function

$$V(w, T-t) = E_t \max_s \int_t^T \tilde{u} \exp(-rz)\, dz \tag{4.1}$$

and applying Bellaman's Principle

$$V(w,t) = E_t \max_s \left\{ \tilde{u} \exp(-rt)\Delta t + V(w+\Delta w, t+\Delta t) \right\} \tag{4.2}$$

But $V(w + \Delta w,\ t + \Delta t) = dV + V(w,t)$

and applying Ito's rule differentiation

$$\begin{aligned} dV &= V_t dt + V_w dw + \tfrac{1}{2} V_{ww} (dw)^2 = \\ &= V_t\, dt + V_w\, dw + \tfrac{1}{2} (\sigma w)^2 V_{ww} dt \end{aligned} \tag{4.3}$$

where again the multiplication rules

$$dt \cdot dt = 0$$
$$dz \cdot dt = 0$$
$$dz \cdot dz = dt$$

have been applied throughout.

Hence we can plugg (4.3) in (4.2)

$$-V_t\, dt = E_t \max_{s(t)} \left\{ \tilde{u} e^{-rt} + V_w g\, dt + \tfrac{1}{2} V_{ww} (\sigma w)^2 dt + V_w \sigma dz \right\} \tag{4.4}$$

where $g = -\beta w + \alpha s + \gamma w$ (4.5)

but taking expectations in RHS (4.4), the term on dz vanishes, and dividing by dt all through we get

$$-V_t = \max_{s(t)} \left\{ \tilde{u} e^{-rt} + V_w g + \frac{1}{2} V_{ww} (\sigma w)^2 \right\} \tag{4.6}$$

If we now define the current maximum value function J, and let $T \to \infty$ as to get rid of the time dependence of J, we can do

$$V = e^{-rt} J \tag{4.7}$$

$$V_w = e^{-rt} J_w \tag{4.8}$$

$$V_t = -re^{-rt} J + e^{-rt} J_t \tag{4.9}$$

and hence plugging (4.7) - (4.9) in (4.6) we get the equation (60) in the text.

APPENDIX 5

We need a closed for solution for S, so we procesed as follows:

Let us guess

$$J = K_1 W - K_2 W^2 + K_o \tag{5.1}$$

then

$$J_w = K_1 - 2 K_2 W \tag{5.2}$$

$$J_{ww} = -2 k_2 \tag{5.3}$$

$$S = 4 \alpha K_1 - 8 \alpha K_2 W \tag{5.4}$$

Therefore, plugging (5. 2) - (5.4) into (60)

$$r(K_o + K_1 w - K_2 w^2) = \Big\{ \tfrac{1}{2} \gamma_n w - \tfrac{1}{2} \psi w^2 - 2 \alpha^2 K_1^2 - 8 \alpha^2 K_2^2 W^2 +$$

$$+ 8 \alpha^2 K_1 K_2 w - \beta K_1 w + 2K_2 \beta w^2 + 4 \alpha^2 K_1^2$$

$$- 8 \alpha^2 K_1 K_2 W - 8 \alpha^2 K_1 K_2 W + 16 \alpha^2 K_2^2 w^2$$

$$+ \gamma_w K_1 - 2 \gamma_w K_2 w - K_2 \sigma^2 w^2 \Big] \tag{5.5}$$

so that the following relations must hold

$$-rK_2 = -\tfrac{1}{2} \psi - 8 \alpha^2 K_2^2 + 2K_2 \beta + 16 \alpha^2 K_2^2 - K_2 \sigma^2 \tag{5.6}$$

$$rK_1 = \tfrac{1}{2}\,\gamma_n + 8\,\alpha^2 K_1K_2 - \beta K_1 - 8\,\alpha^2 k_1 K_2$$
$$-8\,\alpha^2 K_1K_2 - 2\,\gamma_w K_2 \tag{5.7}$$

$$rK_o = -2\alpha^2 K_1^2 + 4\alpha^2 K_1^2 + \gamma_w K_1 \tag{5.8}$$

From (5.6)

$$8\alpha^2 K_2^2 = (\sigma^2 - r - 2\beta)k_2 - \tfrac{1}{2}\psi = 0$$
$$k_2 = \frac{1}{16\alpha^2}\left\{(\sigma^2 - r - 2\beta) \pm \left[(\sigma^2 - r - 2\beta)^2 + 16\alpha^2\psi\right]^2\right\} \tag{5.9}$$

where to ensure $K_2 > 0$ we chosse the positive root K_2.

Now from (5.7)

$$k_1 = \frac{1}{r + \beta + 8\alpha^2 K_2}\left(\frac{1}{2}\gamma_n - 2K_2\gamma_\omega\right) \tag{5.10}$$

Similary

$$K_o = \frac{1}{r}\left(2\alpha^2 K_1^2 + \gamma_\omega K_1\right) \tag{5.11}$$

where the range of γ_ω, γ_n is that wich permits $J_w > 0$.

NOTES

(i) This I shall carry on throughout the analysis. It is fair to do so, because γ_n, γ_w are pure indexes of exogenous shift factors that we can choose to have a well defined problem. Is is however possible to think off corner steady state solutions in wich

$$\beta \gamma_n - 2\psi \gamma_w < 0$$

and hence S*=0, but are not of special interest on this simple example.

(2) Actually from eq. (5) in Appendix 2 we've got

$$W = \frac{\alpha}{\lambda+\beta} S + \frac{1}{\lambda+\beta}\left(\lambda W_c + \gamma_w\right)$$

This the horizontal shift in S is given by

$$dS = -\frac{1}{\alpha}\left(\lambda\, dW_c + d\gamma_w^*\right) =$$

$$= -\frac{1}{\alpha}\left(-\frac{\lambda}{\Delta_c}(\beta+r) + 1\right) d\gamma_w^* =$$

$$= -\frac{1}{\alpha}\left(\Delta_c - \lambda(\beta+r)\right) d\gamma_w^* =$$

$$= -\frac{\lambda}{\alpha\Delta_c}\left(V - (\beta+r)\right) d\gamma_w^*$$

where V is the unstable (positive) eigenvalue of Δ_c, wich is given by

$$V = \frac{1}{2}\left[(a_{11} + a_{22}) + \sqrt{(a_{11}+a_{22})^2 - 4\Delta_c}\right]$$

hence

$$V - (\beta+r) = \frac{1}{2}\left\{ (\partial_{11}-\partial_{22}) + \left[(\partial_{11}-\partial_{22})^2 + 4\partial_{12}\partial_{21}\right]^{1/2}\right\} > 0$$

because the term under the square root is bigger than the term outside.

(3) One could impose consistent conjectural variations CCV as explained in Basar (1985), Perry (1984), Bresnashan (1982). The consideration of CCV however is not yet broadly accepted as a me aningful concept of non cooperative equilibrium. It posses many complications in the solution and it is beyond the scope of this explanatory example.

(4) Actually is easy to see how this is so as

$$\frac{\Delta_c}{\Delta_{OL}} = \frac{-(\beta(\beta+r)+4\alpha^2\psi)}{-(\beta(\beta+r)+2\alpha^2\psi)} < 2$$

(5) ACtually let us claim:

$$\frac{\lambda_{OL}}{\Delta_{OL}} > \frac{2\lambda_c}{\Delta_c}$$

then

$$\frac{\lambda_{OL}}{\Delta_{OL}} > \frac{2\lambda_c}{\Delta_c} \Rightarrow \frac{1}{V_{OL}} > \frac{2}{V_c} \Rightarrow \frac{V_c}{V_{OL}} > 2$$

but

$$V_c = \frac{1}{2}\left\{ r + \left[r^2 + 4\beta(r+\beta) + 16\alpha^2\psi\right]^{1/2}\right\}$$

$$V_{OL} = \frac{1}{2}\left\{ r + \left[r^2 + 4\beta(\beta+r) + 8\alpha^2\psi \right]^{1/2} \right\}$$

and is esay to check that

$2V_{OL} > V_C$ wich contradicts the claim.

Then it must be the case that $\frac{\lambda_{OL}}{\Delta_{OL}} < \frac{2\lambda_C}{\Delta_C} \Rightarrow \left.\frac{dS_{OL}}{d\gamma_n^*}\right|_{t=t_0^+} < \left.\frac{dS_C}{d\gamma_n^*}\right|_{t=t_0^+}$

(6)This similarity of ranking among short and long run effects is intuitively appealing. Under rational expectations, the initial movement in the forward looking variables is a kind of first step of the present value of the total long run change. It is then clear that the bigger the long run response the stronger the sort run one too, in this optimizing models.

(7) If we compare (37) and (53) we see how

$$\frac{dW_{OL}}{d\gamma_w^*} = -\frac{1}{\Delta_{OL}}(\beta+r)$$

$$\frac{dW_F}{d\gamma_w^*} = -\frac{1}{\Delta_F}(\beta+r-\alpha L)$$

$$-\frac{\beta+r}{\Delta_{OL}} = \frac{\beta+r}{\beta(\beta+r)+2\alpha^2\psi} = \frac{1}{\beta+\frac{2\alpha^2\psi}{\beta+r}} <$$

$$< \frac{\beta+r-\alpha L}{\beta(\beta+r-\alpha L)+2\alpha^2\psi} = \frac{1}{\beta+\frac{2\alpha^2\psi}{\beta+r-\alpha L}} = -\frac{\beta+r-\alpha L}{\Delta_F}$$

(8) Recall

$$\left.\frac{dS_{OL}}{d\gamma_n^*}\right|_{t=t_0^+} = \frac{\lambda_{OL}}{\Delta_{OL}} = \frac{1}{V_{OL}}$$

$$\left.\frac{dS_F}{d\gamma_0^+}\right|_{t=t_0^+} = \frac{\lambda_F}{\Delta_F} = \frac{1}{V_F}$$

where again λ and V are the stable (negative) and unstab[le] eigenvalue of the system respectively. But

$$V = \frac{1}{2}\left\{(a_{11}+a_{22}) + \left[(a_{11}+a_{22})^2 - 4\Delta\right]^{1/2}\right\}$$

and it follows from this that

$$V_F = \frac{1}{2}\left\{(r-\alpha L) + \left[(r-\alpha L)^2 - 4\Delta_F\right]^{1/2}\right\} >$$

$$> \frac{1}{2}\left\{r + \left[r^2 - 4\Delta_{OL}\right]^{1/2}\right\} = V_{OL}$$

Therefore the impact effect under (F) is samller than under (OL)

(9) The conditions on $\Gamma(w)$ are, to be twice continuosly differentia[ble] and

$$\lim_{w\to 0}\left(\Gamma'(w)\,\sigma^2 w^2\,\Pi(w)\right) = 0$$

$$\lim_{w\to\infty}\left(\Gamma'(w)\,\sigma^2 w^2\,\Pi(w)\right) = 0$$

but for $\Gamma(w) = w$

$$\Gamma'(w)\sigma^2 w^2 \Pi(w) = m\, w^{-2\theta_1/\sigma^2} \exp\left(-\frac{\theta_2}{\sigma^2 w}\right)$$

and is easy to check that this expression satisfy the conditions above.

(10) (13') can be obtained by proceeding exactly in the same way as we did in the stochastic case to obtain S, K_1, K_2, Furthermore it can be easily checked how (13) and (13') coincide

(11) Is easy to see how

$$E(S) = 4\alpha K_1 - 4\alpha K_2\, E(w) =$$
$$= 4\alpha \left\{ \frac{\beta k_1 - 2k_2 \gamma_w}{\beta + 8\alpha^2 k_2} \right\}$$

This expression is decreasing in σ, and hence as uncertainty increases, the (positive) difference between S an E (S) gets bigger.

(12) Is easy to check how D_1, D_2 are obtained exactly as K_1, K_2 were: An sketch of the process is given below.

Let us guess:

$$J^1 = d_o + d_1 w - d_2 w^2$$

$$r\,(d_o + d_1 w - d_2 w^2) = \Big\{ \tfrac{1}{2}\,\gamma_n\, w - \tfrac{1}{2}\,\psi\, w^2$$

$$-\tfrac{1}{2}\,\alpha^2\,(d_1^{\,2} + 4d_2^{\,2}\,w^2 - 4d_1 d_2 w) - \beta\, d_1 w$$

$$+ 2\,\beta\, d_2 w^2 + 2\,\alpha^2 d_1^{\,2} - 4\,\alpha^2\, d_1 d_2 w$$

$$- 4\,\alpha^2 d_1 d_2 w + 8\,\alpha^2 d_2^{\,2} w^2$$

$$+ \gamma_w d_1 - 2\,\gamma_w d_2 w - \sigma^2 w^2 d_2$$

and hence

$$-rd_2 = -\tfrac{1}{2}\,\psi - 2d_2^{\,2}\,\alpha^2 + 2\,\beta\, d_2 + 8\,\alpha^2 d_2^{\,2} - \sigma^2 d_2$$

$$rd_1 = \tfrac{1}{2}\,\gamma_n + 2\,\alpha^2 d_1 d_2 - \beta\, d_1 - 4\,\alpha^2 d_1 d_2 - 4\,\alpha^2 d_1 d_2 - 2\,\gamma_w d_2$$

so that

$$d_2 = \frac{1}{12\alpha^2}\left\{(\sigma^2 - r - 2\beta) + \left[(\sigma^2 - r - 2\beta)^2 + 12\alpha^2\psi\right]^{1/2}\right\}$$

$$d_1 = \frac{1}{r + \beta + 6\alpha^2 d_2}\left(\frac{1}{2}\gamma_n - 2\,\gamma_w d_2\right)$$

but

$$J_N = J_N^{\,1} + J_N^{\,2} = D_o + D_1 W - D_2 W^2$$

where

$D_o = 2d_o$

$D_1 = 2d_1$

$D_2 = 2d_2$

hence

$$D_2 = \frac{\lambda}{6\alpha^2}\left\{(\sigma^2 - r - 2\beta) + \left[(\sigma^2 - r - 2\beta) + 12\alpha^2\psi\right]^{1/2}\right\}$$

$$D_3 = \frac{\lambda}{r + \beta + 3\alpha^2 D_2}\left(\gamma_n - 2\gamma_w D_2\right)$$

as given in the text.

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